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THE RELATION BETWEEN THE WIEDEMANN EFFECT AND THE JOULE MAGNETOSTRICTION EFFECT

M. E. Fromy

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THE RELATION BETWEEN THE WIEDEMANN EFFECT AND THE JOULE MAGNETOSTRICTION EFFECT*

M. E. Fromy

Engineer, Central Military Telegraphy Agency

ABSTRACT: Wiedemann's magnetic torsion is a special case of Joule's magnetostriction. A cause and effect relation can be established between these two phenomena, provided all the deformations produced in the Joule effect by the inductor field are taken into account.

In Wiedemann's experiments a tube is inserted into a helicoidal field which causes dimensional variations in all directions parallel to the field (the longitudinal Joule Effect) and perpendicular to it (the transverse Joule effect). The study of these two effects shows an asymmetry which results in a twisting of the tube, and leads to an equation which accounts for all the peculiarities of the Wiedemann effect; in particular, it construes the existence of a maximum torsion and an inversion point for some fields different from those which produce some analogous effects in the Joule phenomenon.

The torsion inversion is produced when the coefficients of the transverse and longitudinal Joule effects are equal and of the same sign.

The composite curve of the Wiedemann effect, obtained from those of the two individual Joule effects, is in good accord with the experiment.

1. Previous Work. A number of experimenters have dealt with the magnetic torsion discovered by Wiedemann**. /13***

The majority limited itself to drawing curves of the torsion at angle by varying the metal, its dimensions, the field and the current, etc.

The results obtained are therefore rather confusing and not always in good agreement. The most indicative curves have been provided by the work of Jouaust, which was continued by Pellet, and by that of Williams who has published a whole series of studies on this question. In these he has attempted to establish a relationship between the Wiedemann effect and the Joule magnetostriction effect.

*C.R., t. 181 (Dec. 28, 1925), no. 26.

** See the bibliography at the end of the article.

*** Numbers in the margin indicate pagination in the foreign text.

These different works seem to condense the state of actual knowledge of the subject. We shall recapitulate them briefly in order to elicit from them the general laws, then we shall interpret them and reduce the Wiedemann effect to Joule's fundamental phenomenon, of which it seems to be only a special case.

Jouaust and Pellet used wires 0.2 to 0.3 mm in diameter and about 55 cm long. These wires were attached at their tops and hung vertically, being pulled down by lead weights. They were magnetized longitudinally by a long solenoid and carried a continuous current. The typical results for an iron wire 0.21 mm in diameter and 55 cm long are summed up in Fig. 1; the torsion is measured, in millimeters, by the displacement of a spot on a graduated scale.

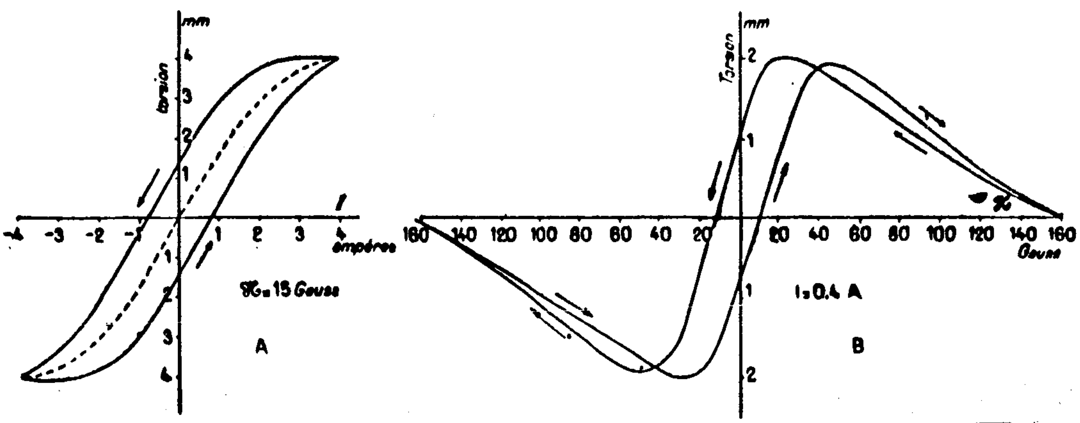

Figure 1. A. Iron Wire. Diameter: 0.021; Length, 55 cm.

Williams' tests were made on tubes 80 cm long and 0.5 to 1 mm average radius.

These tubes were magnetized longitudinally by a long solenoid; the excitaion current was transmitted either through the tube itself or an through an insulated wire placed along its axis.

Williams measured simultaneously (on each of the tubes) the Wiedemann effect and the longitudinal Joule effect. The curves obtained are all presented in the /14
form of Fig. 2 which shows the results for a steel tube 80.2 cm long, with an inside diameter of 0.1538 cm and an outside diameter of 0.2386 cm.

Finally, Williams has studied simultaneously the transverse and longitudinal Joule effects on samples of iron, nickel, and cobalt. The results obtained are presented in the form of Figs. 3 and 4, which show the results for an iron tube 30 . 74 cm long and for a nickel tube.

All these experiments lead to the following conclusions:

1. In a magnetic field, a magnetic metal undergoes length changes in the direction of the field (the longitudinal Joule effect) and in all directions perpendicular to the field (the transverse Joule effect).

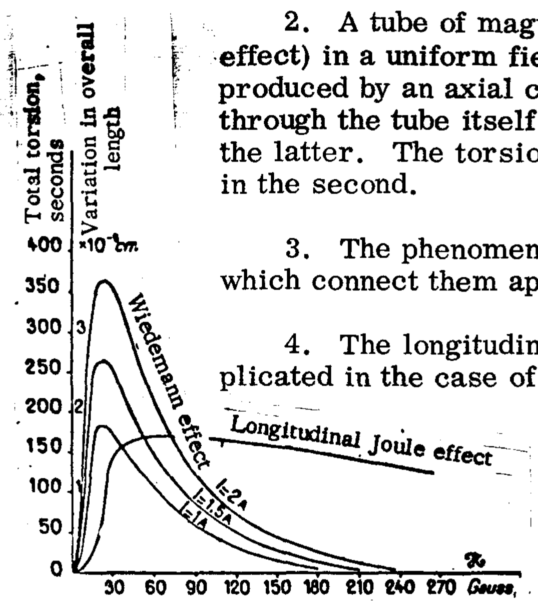

Figure 2. Steel Tube. Average Radius: 0.098 cm; Length: 80.2 cm.

2. A tube of magnetic metal undergoes torsion (The Wiedemann effect) in a uniform field parallel to its axis and in a circular field produced by an axial current. The axial current can travel either through the tube itself or an insulated wire placed along the axis of the latter. The torsions are merely weaker in the first case than in the second.

3. The phenomena seem of the same nature but the relationships which connect them appear complex.

4. The longitudinal and transverse Joule effects are very complicated in the case of iron and steel. In weak fields one observes first a longitudinal expansion and then a transverse contraction; in high strength fields, the reverse is observed. The curves show some inversion points which are not necessarily common for the two effects.

In the case of nickel, a longitudinal contraction and a transverse expansion are always observed; the phenomena seem much simpler.

5. The rules governing the Wiedemann effect may be summed up as follows:

(a) For a constant field, the torsion increases at first proportionally to the current up to a certain value, after which it seems to tend toward a limit or a maximum. /15

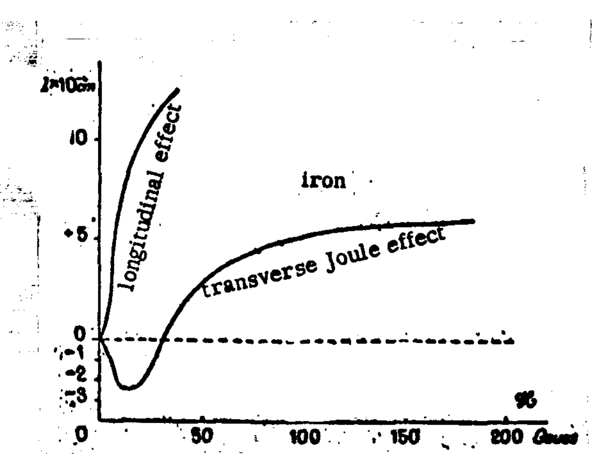

Figure 3.

(b) For a constant current, the torsion increases at first proportionally to the field so long as the latter is weak; then it passes through a maximum and decreases. The decrease, rapid at first, gradually slackens.

In the case of iron, the decrease becomes zero between 150 and 200 gauss, and then reverses. Both the increase and the decrease are less rapid in the case of nickel; moreover, there is no inversion point, the torsion is always of the same sign and decreases continually and very slowly.

(c) The maximum torsion is produced at a certain field strength which is independent of the current traversing the wire. Furthermore, this field strength differs from that which corresponds to the maximum longitudinal Joule effect.

(d) The inversion point in the case of iron seems to have no relation to the analogous points encountered in the case of the two Joule effects.

(e) The torsions are of opposite sign for iron and nickel so long as the field remains less than the inversion value of iron.

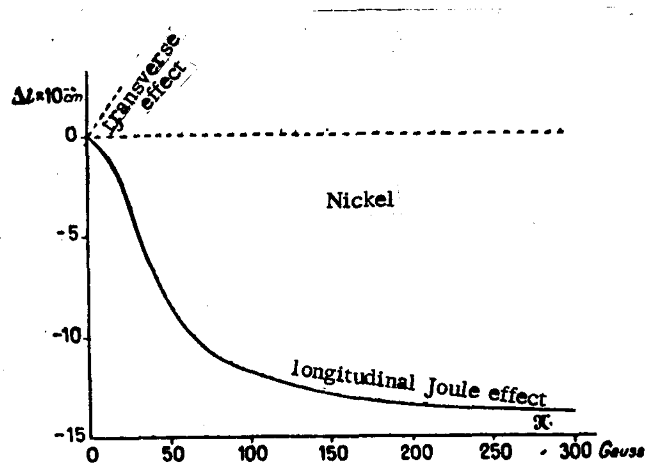

Figure 4.

6. All magnetostriction phenomena (Joule and Wiedemann) exhibit hysteresis.

Alone among all the authors, Williams has tried, not so much to establish a general theory of magnetostriction phenomena, but to relate magnetic torsion to the Joule effects of which it seems to be a direct consequence or a special case. Williams' reasoning was the following:

Consider a tube inside of a magnetic metal with a length L an average radius r (Fig. 5) placed in a uniform field H parallel to its axis and to a circular field C produced by an axial current I.

These two fields have a resultant $\mathcal{H}$ which makes an angle φ with H so that

$$\tan\varphi = \frac{C}{H}.$$

It follows that the length variations due to the Joule effect will occur along a helix slanting at an angle φ to the generatrices of the cylinder. These length variations involve a displacement AB from the end A of the helix; if J designates the coefficient of the Joule effect, this displacement is expressed by $\frac{JL}{\cos\varphi}$. There are two components, one parallel to the axis of the tube and corresponding to the variation in the axial length, the other tangent to the tube and characterizing a rotation by the angle:

$$\theta = \frac{JL}{r}\tan\varphi.$$

Expressing $\tan\varphi$ as a function of H and I, one obtains finally:

$$\theta = \frac{2JL}{r^2}\cdot\frac{I}{H}. \quad (1)$$

This equation takes into account the general behavior of the phenomenon from the point of which H becomes sufficiently large but is quite inaccurate at low values of H. Specifically, if the axial field is zero, we have the simple Joule effect produced by the circular field C, and the magnetic torsion should become zero. The Williams equation indicates, to the contrary, that it becomes infinite. /16

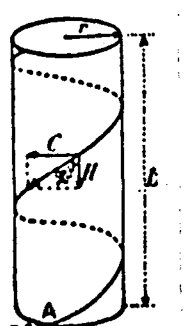

Figure 5.

Furthermore, this equation does not indicate a torsion maximum; one can only foresee qualitatively its existence in speculating on the variation of J.

It indicates that the torsion should always be proportional to I whereas, in practice, a limit or maximum is found in certain cases.

Finally, the formula is quite useless in interpreting the torsion reversal in the case of iron, at a field value different from those which correspond to the inversion points of Joule effects.

The Williams theory seems, therefore, to be very approximate. We wanted, therefore, to establish between the Joule and the Wiedemann effects, a relationship resting on a more solid basis.

To do this, it seemed indispensible to take into account not only the longitudinal Joule effect but also the transverse Joule effect, which is of the same order of magnitude. This is what we are going to do below, our purpose being not to establish a complete theory of the magnetostriction phenomena, but simply to find a relation between the Joule and the Wiedemann effects and to derive an explanation of the latter by utilizing the experimental curves of the two former effects.

2. Fundamental Phenomenon. If a magnetic metal is placed in a uniform field $\mathcal{H}$, H varies in length in directions parallel and perpendicular to the field.

Parallel to the field there is a variation in the quantity ρ which we shall consider positive if a contraction occurs, and negative in the case of an expansion.

In all directions perpendicular to $\mathcal{H}$ there is a length variation in the quanitity ρ', to which we assign the same sign convention as ρ.

This being so, let us imagine an infinite flat metal plate of a thickness ϵ and located within a uniform field $\mathcal{H}$ parallel to its surface.

We then have:

1. a contraction ρ parallel to $\mathcal{H}$,

2. a contraction ρ' perpendicular to $\mathcal{H}$ in the plane of the plate,

3. a contraction ρ' in the direction of the thickness, i. e., a variation $\rho'\epsilon$ of the thickness.

Suppose now that this plane is rolled into a cylinder, while the field $\mathcal{H}$ remains parallel at every point to the surface element, therefore in a plane tangent to the cylinder; consequently, the field becomes helical or circular (in the case in which the axis of the cylinder is perpendicular to the field).

There are again some contractions ρ parallel to the field at each point and some contractions ρ' perpendicular to this field. However, since the upper part of the tube will be fixed in a rigid support, all the points on the rigidly held support circumference will be constrained not to leave this circumference. The contractions ρ and ρ' will therefore produce some slipping parallel to the plane of the support, and this will manifest itself as a torsion of the tube.

Thus one arrives at the Wiedemann phenomenon starting the Joule effects.

We shall first study the deformations of an infinite flat plate; then we shall roll the plate to form a cylinder and we shall attempt to deduce the deformation of the tube from those of the plate.

3. The Deformation of a Plate. Let us imagine a magnetic metal plate with a thickness ε in the form of an infinite plane which we take as the plane of the figure (Fig. 6).

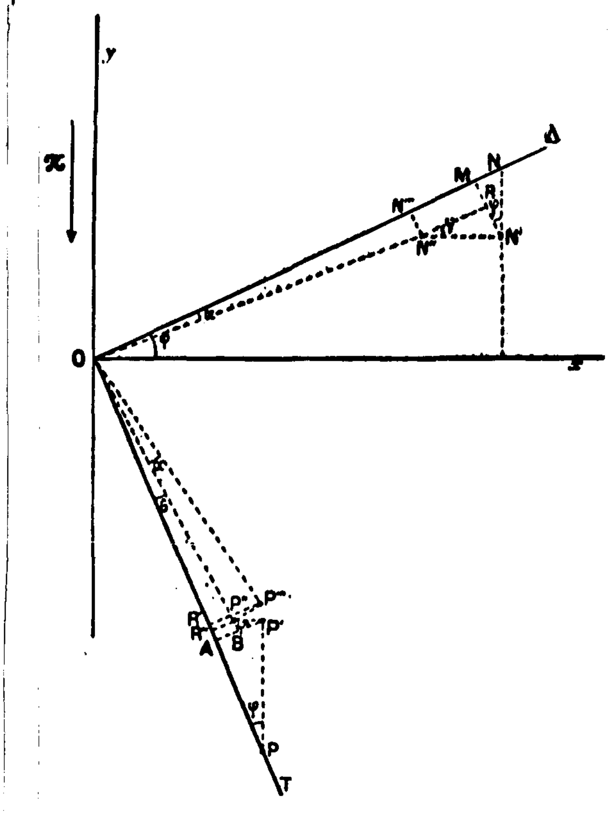

Figure 6

Let us draw on this plate a straignt line Δ and assume, a priori, that the points of this line are required to to remain on it during all deformations of the plate.

Let us consider a point O on this line as the origin of a system of coordinates and draw two coordinate axes, Oy and Ox, making an angle φ with Δ. /17

Futhermore, let there be another point N on Δ, with coordinates of x and y.

Now let us imagine that we place the plate in a uniform field of strength and parallel to the plane of the figure. For the sake of specificity, let the field be parallel to the Oy axis.

We shall then have:

1. a contraction ρ parallel to Oy which will bring point N to N' so that $NN' = \rho y$,

2. a contraction ρ' parallel to Ox which will bring point N' to N'' so that $N'N'' = \rho' x$,

3. a contraction ρ' in the direction of the thickness of the plate, or a variation in the thickness $\rho'\varepsilon$.

Point N has been superposed on N''; since, by definition, it may not leave the straight line which we assumed is fixed in space, we must rotate the entire figure, including the coordinate axes, by an angle α around O, so as to superpose N'' onto N''', (which is on Δ).

Thus the actual displacement of the point N is NN''' on the straight line Δ, which corresponds to a contraction $\rho_\Delta = \frac{NN'''}{ON}$ parallel to Δ.

Assuming rather small deformations of the plate so as to be able to neglect terms of the second order, the various components can be easily calculated.

To do that, let us construct the perpendiculars N'M to Δ and N''R to N'M; the solution of the triangles MNN' and N''RN', on the one hand, and of the curvilinear quadrilateral RMN'''N'' (assumed to be a rectangle) on the other, gives:

$$\rho_\Delta = \frac{NN'''}{ON} = \frac{RN' + NM}{ON} = \frac{N'N'' \cos\varphi + NN' \sin\varphi}{ON},$$

$$\alpha = \frac{N'N'''}{ON} = \frac{N'M - N'R}{ON} = \frac{NN' \cos\varphi - N'N'' \sin\varphi}{ON}.$$

which finally gives

$$\rho_\Delta = \rho \sin^2\varphi + \rho' \cos^2\varphi;$$
$$\alpha = (\rho - \rho') \sin\varphi \cos\varphi.$$

Now let us consider another point P on the plane. Prior to the deformation of the plane, P is located on line OT perpendicular to Δ and has the coordinates of x' and y'.

Under the action of magnetostriction effects, point P underposes three consecutive displacements:

1. a displacement $PP' = \rho y'$, due to the contraction ρ;

2. a displacement $P'P'' = \rho' x'$, due to the contraction ρ';

3. a rotation by the angle α, corresponding to the displacement N''N''' of the point N, which brings point P'' to P'''.

The resultant displacement of PP''' corresponds to: /18

1. a contraction $\rho_T = \frac{PR'}{OP}$ parallel to OT;

2. a slip R'P''' parallel to Δ that is a unit slip.

$$g = \frac{R'P'''}{OP}.$$

By dropping perpendicular P'A onto OT and P''B onto P'A, reasoning as before and making points R' and R'' coincide (these are very close to each other if α is small). one finds:

$$\rho_T = \frac{PA + AR'}{OP} = \frac{PA + AR''}{OP} = \frac{PP' \cos\varphi + P'P'' \sin\varphi}{OP};$$

$$g = \frac{OP(\alpha + \beta)}{OP} = \alpha + \beta = \alpha + \frac{AB}{OP} = \alpha + \frac{PP' \sin\varphi - P'P'' \cos\varphi}{OP}.$$

which finally gives:

$$\rho_T = \rho' \sin^2\varphi + \rho \cos^2\varphi;$$
$$g = 2(\rho - \rho') \sin\varphi \cos\varphi.$$

Conclusion. To conclude this study let us pose the problem in another form.

Let us consider an infinite flat plate of a magnetic metal with a thickness ϵ.

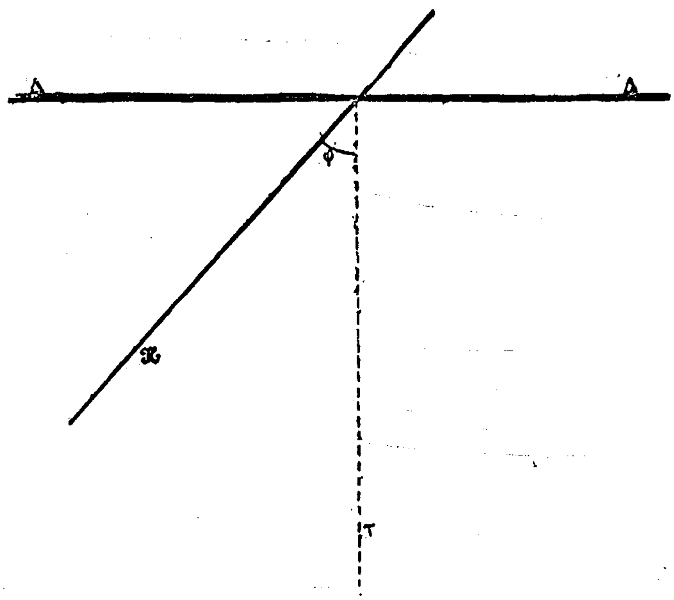

Figure 7

Let us imagine a straight line Δ situated in this plane; all the points of Δ are constrained to shift along this line, whatever happens to the plate. Let us place this plate in a uniform field of strength $\mathcal{H}$, parallel to the plate and making an angle φ with the direction T perpendicular to Δ.

The deformations due to magnetostriction are represented in the present case by:

1. a contraction along Δ,

$$\rho_\Delta = \rho \sin^2\varphi + \rho' \cos^2\varphi;$$

2. a contraction along the T (the perpendicular to Δ)

$$\rho_T = \rho \sin^2\varphi + \rho \cos^2\varphi;$$

3. a unit of the various layers of the metal parallel to Δ: /19

$$g = 2(\rho - \rho') \sin\varphi \cos\varphi;$$

4. a contraction ρ' in the direction of the thickness

4. Deformation of a Cylinder. Consider a cylindrical tube made of a magnetic substance. Let L be the length; r average radius; and ϵ, the thickness of the cylinder tube wall.

Let us assume, for simplicity, that the thickness ε is small compared to the average radius r.

Then, let us assume that this cylinder is fixed at its upper end to some support and let us place it in two continuous fields:

1. a uniform field of the strength H, parallel to the axis of the cylinder;

2. A circular field C produced by a current I passing through an insulated conductor arranged along the axis of the tube.

If ε is small compared to r, we can assume that the circular field is the same at all points in the metal and equal to

$$C = \frac{2I}{r}.$$

Therefore, at any point M of the tube, the metal is simultaneously affected by the two fields H and C, which result in a combined field

$$\mathcal{H} = \sqrt{H^2 + C^2}$$

making an angle φ with the corresponding generatrix of the cylinder such that

$$\tan \varphi = \frac{C}{H}.$$

The tube is deformed by this field contracting or expanding along the direction of the resultant field $\mathcal{H}$ and along the all perpendicular directions to that field.

Furthermore, all the points which are located in the circumference constrained by the support must remain in the plane of this circumference.

Under these conditions we have a situation identical to that we have encountered in the case of the flat plate.

The circumference encased by the support plays the role of the straight line Δ, and the resultant field $\mathcal{H}$, that of the uniform inducing field.

The actual case can be deduced from the preceding case by rolling the metallic plate into a cylinder around an axis parallel to the plane of the plate and perpendicular to Δ; the straight line Δ thus becomes a circular figure coinciding with the cylinder support.

The resulting deformations of the tube will be:

1. a radial deformation caused by the contraction along the circumference of the tube:

$$\rho_r = \rho \sin^2 \varphi + \rho' \cos^2 \varphi;$$

2. an axial deformation:

$$\rho_a = \rho' \sin^2 \varphi + \rho \cos^2 \varphi;$$

3. a slip parallel to the circumference of the tube support, that is, a unit torsion:

$$\theta = \frac{2(\rho - \rho')}{r} \sin\varphi \cos\varphi;$$

4. a thickness variation ρ'.

Substituting for φ its value as a function of H and C, we finally obtain: /20

$$\rho_r = \rho \frac{C^2}{H^2 + C^2} + \rho' \frac{H^2}{H^2 + C^2}; \tag{2}$$

$$\rho_a = \rho' \frac{C^2}{H^2 + C^2} + \rho \frac{H^2}{H^2 + C^2}; \tag{3}$$

$$\theta = 2 \frac{\rho - \rho'}{r} \frac{HC}{H^2 + C^2}. \tag{4}$$

The only equation which interests us at the moment is the third one, which allows us to compute the torsional angle, that is, the Wiedemann effect. We shall now deal only with that effect.

<u>Discussion.</u> The torsional angle equation

$$\theta = 2 \frac{\rho - \rho'}{r} \frac{HC}{H^2 + C^2}$$

makes it possible to take into account all the features of the Wiedemann effect, contrary to the approximate equation developed by Williams.

Without extending the discussion further, we see immediately that the torsion is zero when one or the other of the two fields becomes zero; in which case only the contractions ρ_r and ρ_a, due to the simple Joule effects, remain. Between these two extremes, we can find a torsional maximum, the equation for which we shall investigate below, and which is independent of the maximum Joule effect.

Thus the torsion can become zero even if the inducing fields and the Joule effect are not zero. For this to occur, it is sufficient that the coefficients ρ and ρ' of the two Joule effects be equal and of the same sign, as can occur in the case of iron (see the curves of Fig. 3).

The direction of the torsion changes with the sign of the factor $(\rho - \rho')$, which explains why the rotation of the tube end has a different sign for iron and nickel.

Finally, the torsion inverts when one of the factors H or C changes sign.

A complete discussion of Eq. 4 is complex. It becomes, however, very simple when one of the fields is weak compared to the other.

First let us study the case of $C < H$.

In this case C^2 is negligible compared to H^2 and the expression for θ becomes

$$\vartheta = 2\frac{\rho - \rho'}{r}\frac{C}{H}. \qquad (5)$$

In addition, ρ and ρ', which are functions of the resultant field $\varkappa = \sqrt{H^2 + C^2}$, can be regared simply as functions of H.

This gives, first of all, that the torsion is a linear function of the weaker field C, so long as C^2 can be neglected compared to H^2.

The torsion variation as a function of the stronger field H is more complicated; it varies as the function

$$y = \frac{\rho - \rho'}{H}.$$

The functions ρ and ρ' are not known analytically but are given by the experimental curves.

Imagine that they are known and assume they are represented in the form of Fig. 8 (the solid lines).

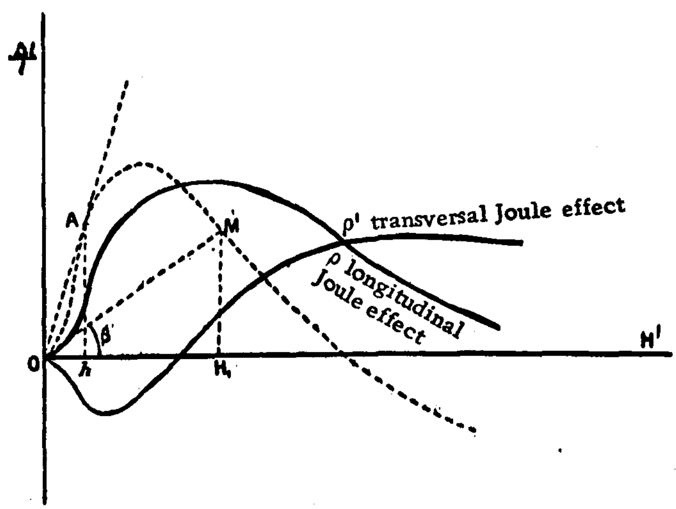

Figure 8

The function $\rho - \rho'$ is represented by the dashed curve whose ordinates are the algebraic difference of the ordinates of the curves for ρ and ρ', and the function $y = \frac{\rho - \rho'}{H}$, will be represented by the tangent to the angle β, that is, the slope of the cord OM.

One can then write:

$$\vartheta = 2\frac{C}{r}\tan\beta. \qquad (6)$$ /21

The discussion of the change in ϑ as a function of the field H at constant C then becomes very simple.

At weak H, the torsion obeys Eq. 4, that is, it varies approximately proportionally to H.

When C^2 becomes negligible compared to H^2, the torsion is governed by Eq. 6. It continues to increase rapidly, passes through a maximum H=h (at which the cord becomes tangent at A on the dashed curve); then it decreases, first rapidly, then slowly. It becomes zero when H reaches the value at which $\rho = \rho'$, and then inverts.

The torsion maximum occurs for the H value which corresponds to the maximum of tan β; this value which does not depend on C.

Thus far we have assumed that the circular field C was weak compared to H, but the preceding reasoning applies without modification to the case in which H is weak compared to C, the roles of the two fields being simply reversed.

In this case, θ is given by the equation

$$\theta = 2\frac{H}{r}\tan\beta \tag{7}$$

It follows that the torsion will be proportional to H and will vary with the function C as

$$\tan\beta = \frac{\rho - \rho'}{C}.$$

Finally, if the fields H and C are of the same order of magnitude, one must use the complete Eq. (4), which is difficult to discuss; but in practice the region of the curve in which the magnitudes of the two fields are close is rather short so that one can draw it by extrapolation between the two portions of the curves governed by Eqs. (6) and (7).

5. Comparison with the Experiment. A. Variation with a Constant Field and a Variable Current.

In Williams' experiments, the circular field C was produced by a current of the order of several amperes traveling along the axis of a tube with an average radius of approximately 1 mm. It follows that the field $C = \frac{2I}{10r}$ was the order of several gauss. However, the field H varied form 0 to 200 or 300 gauss, and greatly exceeded C over almost the entire length of the curve. The rule for the Wiedemann effect then is given by the equation: /22

$$\theta = \frac{4}{10}\frac{I}{r^2}\tan\beta. \tag{8}$$

This equation shows that, for a constant field, the torsion is proportional to the current. This specifically, is the case for the maxima, as Fig. 2 indicates. The information in Fig. 2 provides the following table:

TABLE 1

I	$\frac{\theta}{I}$
1	180
1.5	180
2	180

On the other hand, in their tests with a constant field and a variable current, Jouaust and Pellet used fine wires magnetized by weak fields H and strong currents.

In the case of Fig. 1 A, the wire used was 0.021 cm in diameter and carried a current of several amperes which created a field C of several tens of gauss at the periphery of the tube. The H field was only about 15 gauss.

The torsion for a constant field and a varible current, governed at first by Eq. (8) when the current is weak, starts to obey Eq. (7) when C becomes greather than H.

It turns out in this case that the torsion is no longer proportional to I at large values of I; it tends toward a maximum which occurs at the value of I which gives a maximum tan β.

The C value corresponding to this maximum must be the same as that of H which corresponds to the maximum in the curve for a constant current and a variable field (Fig. 1 B); it should fall between 30 and 40 gauss.

Now, curve A of Fig. 1 shows that the maximum occurs in the vicinity of 4 amps, that is, for a value of C, at the tube peripherey of:

$$C = \frac{2 \times 4}{10 \times 0.021} = 38 \text{ gauss},$$

this C is of the same order as that cited before (we assumed here that the wire behaves as an infinitely narrow tube consisting primarily of a very thin envelope), the internal layers serving only as conductors of the current.

B. The Variation with a Constant Current and a Variable Field.

In this case C is weak compared to H and the Wiedemann phenomenon obeys the equation:

$$\theta = \frac{4'}{10} \frac{I}{r^1} \tan\beta.$$

Without stating any hypotheses, this equation shows that the torsion passes through a maximum at that value of H which makes tan β maximum and which does not depend on I.

All the maxima should then be located on the same vertical, as the curves in Fig. 2 indicate.

To the complete discussion, we would have to know the two curves for the Joule effect and that for the Wiedemann effect on the same sample. Williams' experiments do not provide enough material for such a discussion.

But one can attempt a synthesis starting from the curves of Fig. 3. for an iron cylinder of length L = 30.7 cm. The transverse Joule effect curve is available up to 200 gauss; that of the longitudinal effect stops at 50 gauss; in order to be able to discuss it, we assume that it has the shape indicated by dashes (Fig. 9). This seems likely, given the usual behavior of the Joule effect in iron.

Let us draw the dot-dash curve whose ordinates are the algebraic difference of the ordinates of the two first curves; this curve represents the function:

$$y = (\rho - \rho')\,L,$$ /23

for iron.

Let us now imagine a tube constructed of the same iron and of the same length (L = 30.7 cm) with an average radius of r = 0.1 cm, which is assumed to be large compared to the thickness ε.

The general expression for the overall torsion of a tube in the Wiedemann effect is

$$\alpha = \theta L = \frac{4}{10}(\rho - \rho')\,L\,\frac{I}{r^2}\cdot\frac{H}{H^2 + \frac{4I^2}{100r^2}},$$

or

$$\alpha = \frac{4}{10}\,y\,\frac{I}{r^2}\cdot\frac{H}{H^2 + \frac{4I^2}{100r^2}}.$$

If we make, for example, I = 1 amp, and r = 0.1 cm, we have

$$\alpha = 40\,\frac{H}{H^2 + 4}\,y.$$

where y is a function of the resultant field

$$\mathcal{H} = \sqrt{H^2 + \frac{4I^2}{100r^2}}.$$

This equation allows us to draw an assumed curve for the Wiedemann effect for the case of iron (Fig. 9).

This curve has the same shape as curves obtained directly by experiment.

It exhibits a maximum at 8 gauss which corresponds to a total torsion of 75 sec.

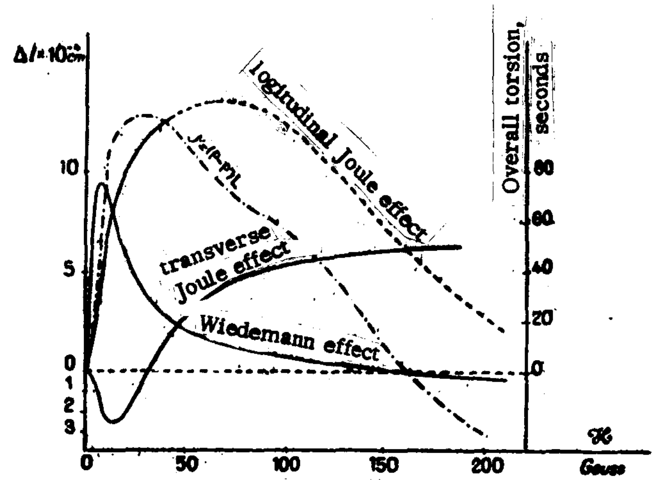

Figure 9.

If the tube were 80 cm long instead of 30.7, the torsion would be 192 sec, a number very close to that in Fig. 2 (180 sec).

There is again a decrease, first rapid, then slow; at a stronger field, the torsion becomes zero and inverts. The inversion point occurs in the vicinity of 160 gauss.

In the case of a nickel tube, one could reason in the same fashion, starting from Fig. 4, had Williams given the curve for the transverse effect. A plot analogous to that in Fig. 9 would have been obtained, but the variations would be slower; in addition there would not have been an inversion point. This is exactly what we find by experiment. /24

6. Conclusion. The formula established above represents rather well the behavior of Wiedemann's magnetic torsion from the qualitative point of view, and allows this torsion to be considered as a simple case of magnetostriction.

Specifically, it takes into account:

1. the proportionality of the torsion to the current so long as the circular field remains smaller than the axial field; in the opposite case, the behavior ceases to be linear, and the torsion reaches a maximum;

2. the existence of a maximum torsion, for a constant current and a weak field, whose strength differs from that inducing the corresponding Joule effects;

3. the existence, in the case of iron, of an inversion point at a field strength at which the Joule effects are not zero.

The quantitative verifications cited above would seem satisfactory, but they could be merely simple coincidences; one could only judge their value by comparing them with the curves of the two Joule effects and the Wiedemann effect surveyed simultaneously in the same tube or sample of iron, nickel, or cobalt.

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